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一、$E_1(s)=\frac{1}{s^2}$ $R_1(s)=\frac{1}{s+a}$ $H(s)=\frac{R_1(s)}{E_1(s)}=\frac{s^2}{s+a}$

$R_2(s)=E_2(s)\cdot H(s)=s-a+\frac{a^2}{s+a}$ $r_2(t)=\delta'(t)$

$-a\delta(t)+a^2e^{-at}u(t)$

二、1 由题可知特征方程为：$\lambda^2+6\lambda+5=0$

$\lambda_1=-1$ $\lambda_2=-5$ ∴

① 求 $r_{zi}(t)$

$r_{zi}(t)=A_1e^{-t}+B_1e^{-5t}$

$\begin{cases}A_1+B_1=r(0_-)=r(0_-)=1\\ -A_1-5B_1=r'(0_-)=r'(0_-)=2\end{cases}$ $A_1=\frac{7}{4}$ $B_1=-\frac{3}{4}$

$r_{zi}(t)=\frac{7}{4}e^{-t}-\frac{3}{4}e^{-5t}$

② 求 $r_{zs}(t)$

因 $e(t)=t^2u(t)$ 可得方程：

$\frac{d^2r(t)}{dt^2}+6\frac{dr(t)}{dt}+5r(t)=(2t+3t^2)u(t)$

特解 $r^*=at^2+bt+c$ 代入方程

$2a+12at+6b+5at^2+5bt+5c=2t+3t^2$

$a=\frac{3}{5}$ $b=-\frac{26}{25}$ $c=\frac{6}{125}$

$r_{zs}(t)=A_2e^{-t}+B_2e^{-5t}+\frac{3}{5}t^2-\frac{26}{25}t+\frac{6}{125}$

没有$\delta(t)$及其导数 ∴ $r(t)$在0处连续

$\begin{cases}A_2+B_2+\frac{6}{125}=0\\ -A_2-5B_2-\frac{26}{25}=0\end{cases}$ 得 $A_2=\frac{25}{125}=\frac{1}{5}$ $B_2=\frac{-31}{125}$

$r_{zs}(t)=\frac{1}{5}e^{-t}-\frac{31}{125}e^{-5t}+\frac{3}{5}t^2-\frac{26}{25}t+\frac{6}{125}$

$r(t)=\underbrace{\frac{39}{20}e^{-t}-\frac{499}{500}e^{-5t}}_{\text{自由响应}}+\underbrace{\frac{3}{5}t^2-\frac{26}{25}t+\frac{6}{125}}_{\text{强迫响应}}$

2. 由题

$\frac{d^2r(t)}{dt^2}+6\frac{dr(t)}{dt}+5r(t)=\delta'(t)+3\delta(t)$

利用冲激函数匹配法求 $r(t)$ $r'(t)$

设 $\begin{cases}r''(t)=a\delta'(t)+b\delta(t)+cu(t)\\ r'(t)=a\delta(t)+bu(t)\\ r(t)=cu(t)\end{cases}$

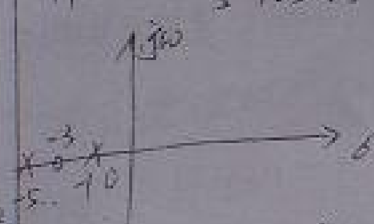
$a=1$ $b+6a=3$ $b=-3$

设 $h(t)=Ae^{-t}+Be^{-5t}$

则 $\begin{cases}A+B=a=1\\ -A-5B=-3\end{cases}$ $A=\frac{1}{2}$ $B=\frac{1}{2}$

$h(t)=\frac{1}{2}e^{-t}+\frac{1}{2}e^{-5t}$

3. $H(s)=\frac{s+3}{s^2+6s+5}=\frac{s+3}{(s+1)(s+5)}$



极点均位于s平面左半面 ∴ 系统稳定

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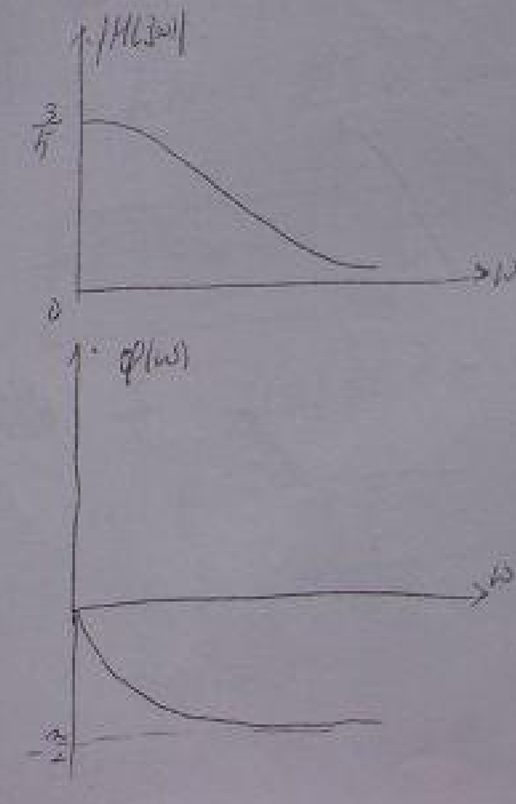
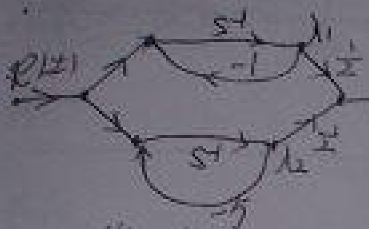


h. $H(s)=\frac{s+3}{(s+1)(s+5)}=\frac{\frac{1}{2}}{s+1}+\frac{\frac{1}{2}}{s+5}$



$\begin{cases}\lambda_1'(t)=\lambda_1(t)+e(t)\\ \lambda_2'(t)=\lambda_2(t)+e(t)\end{cases}$ $\quad r(t)=\frac{1}{2}\lambda_1(t)+\frac{1}{2}\lambda_2(t)$

$\begin{bmatrix}\lambda_1'(t)\\ \lambda_2'(t)\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}\lambda_1(t)\\ \lambda_2(t)\end{bmatrix}+\begin{bmatrix}1\\ 1\end{bmatrix}e(t)$

$r(t)=\left(\frac{1}{2},\frac{1}{2}\right)\begin{bmatrix}\lambda_1(t)\\ \lambda_2(t)\end{bmatrix}$

$A=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\quad B=\begin{bmatrix}1\\ 1\end{bmatrix}\quad C=\left(\frac{1}{2},\frac{1}{2}\right)\quad D=0$

二、解：

$r(t)=e(t)*h_1(t)*h_1(t)*h_2(t)$

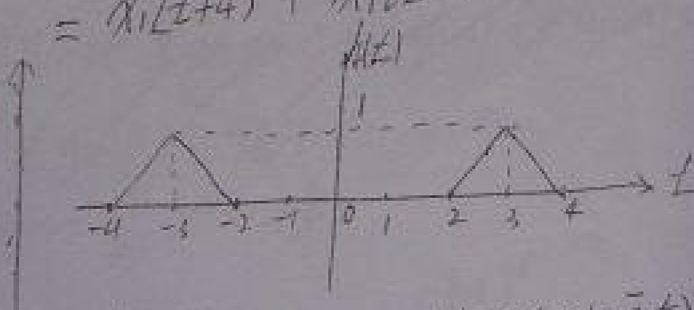
$h(t)=\delta(t)*h_1(t)*h_1(t)*h_2(t)$

$=[h_1(t)*h_1(t)]*[\delta(t+4)+\delta(t-2)]$

设 $h_1(t)*h_1(t)=t\,u(t)+(2-t)[u(t)-u(t-1)]$

$h(t)=x_1(t)*[\delta(t+4)+\delta(t-2)]$

$=x_1(t+4)+x_1(t-2)$



$h(t)=(t+4)[u(t+4)-u(t+3)]+(-2-t)[u(t+3)-u(t+2)]+(t-2)[u(t-2)-u(t-3)]+(-t+4)[u(t-3)-u(t-4)]$

$\int_{-\infty}^{+\infty} h(t)e^{-j\omega t}dt$

$=\int_{-4}^{-3}(t+4)e^{-j\omega t}dt+\int_{-3}^{-2}(-2-t)e^{-j\omega t}dt+\int_{2}^{3}(t-2)e^{-j\omega t}dt+\int_{3}^{4}(4-t)e^{-j\omega t}dt$

$=-\frac{(e^{-j\omega}-1)^2}{\omega^2}\left(e^{+3j\omega}+e^{-3j\omega}\right)$

直接利用 $H_1(j\omega)\cdot H_1(j\omega)\cdot H_2(j\omega)$

四、

1.

$f_T(t)=f(t)\cdot\delta_T(t)=f(t)\sum_{n=-\infty}^{+\infty}\delta(t-nT_1)$

$F_T(\omega)=\frac{1}{2\pi}F(\omega)*\delta_T(\omega)$

$=\frac{1}{2\pi}F(\omega)*\left(\frac{2\pi}{T_1}\right)\sum_{n=-\infty}^{+\infty}\delta(\omega-n\omega_1)\quad \left(\omega_1=\frac{2\pi}{T_1}\right)$

$=\frac{1}{T_1}\sum_{n=-\infty}^{+\infty}F(\omega-n\omega_1)$

2.

设 $|\omega|>\omega_m$ 时 $|F(\omega)|=0$

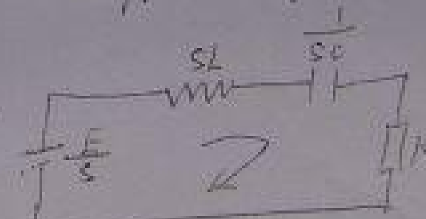
则无失真恢复 $F(\omega)$ 的条件是 $\omega_1\geq\omega_m$

所通过滤波器的特性

频域：$\begin{cases}H(j\omega)=K & |\omega|\leq\omega_m\\ \quad 0 & |\omega|>\omega_m\\ \varphi(\omega)=-\omega t_0\end{cases}$

时域：$h(t)=\frac{\omega_m}{\pi}Sa[\omega_m(t-t_0)]$

五、



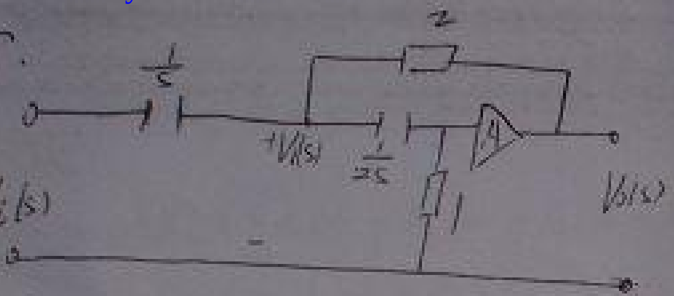
s域：$\frac{E}{s}=I(s)\left(sL+\frac{1}{sC}+R\right)$

$I(s)=\frac{E}{s}\cdot\frac{1}{s+\frac{2}{s}+2}$

$=\frac{E}{s}\cdot\frac{s}{(s^2+2s+2)}$

$=\frac{E}{(s+1)^2+1}$

六1.



1. 由题:

$$\begin{cases} \dfrac{V_i(s)-V_1(s)}{\frac{1}{s}} = \dfrac{V_1(s)-V_0(s)}{2} + \dfrac{V_1(s)}{\frac{1}{2s}+1} \\ \dfrac{V_1(s)}{\frac{1}{2s}+1} A = V_0(s) \end{cases}$$

得: $\dfrac{V_0(s)}{V_i(s)} = \dfrac{s}{\frac{4s^2+2s+2s+1+(\frac{1}{A}-\frac{1}{2})\cdots}{4As}}$

$$= \frac{4As^2}{4s^2+(8-2A)s+1}$$

若要系统稳定 则 $8-2A>0$ $A<4$

2. $A=4$ 时

$H(s) = \dfrac{16s^2}{4s^2+1} = 4 - \dfrac{4}{4s^2+1} = 4 - \dfrac{1}{s^2+\frac{1}{4}}$

$h(t) = 4\delta(t) - (\sin\frac{1}{2}t)u(t)$

七.

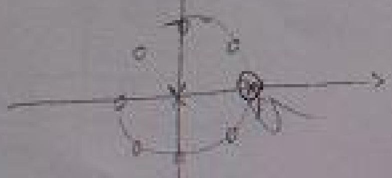
1. ~~$y(n)$ 才 $a^6 y(n-1) + a^5 y(n-2) + a^4 y(n-3) + a^3 y(n-4) + a^2 y(n-5) + a y(n-6)$~~

$y(n) = \sum_{k=0}^{7} a^k x(n-k)$

2. $H(z) = \sum_{k=0}^{7} a^k z^{-k} = \dfrac{1-(az^{-1})^8}{1-az^{-1}}$

3. $h(n) = a^n[u(n) - u(n-8)]$

4.



八.

$y(n) = h(n) * x(n)$

利用乘法原理

		1	a	a^2	a^3	a^4	a^5	a^6	a^7
							1	1	1
		1	a	a^2	a^3	a^4	a^5	a^6	a^7
	1	a	a^2	a^3	a^4	a^5	a^6	a^7	
1	a	a^2	a^3	a^4	a^5	a^6	a^7		

可得 $y(n) = \{1, a+1, a^2+a+1, a(a^2+a+1), a^2(a^2+a+1), a^3(a^2+a+1), a^4(a^2+a+1), a^5(a^2+a+1), a^6+a^7, a^7\}$ ($n=0$)

九.

$H(z) = \dfrac{1-3z^{-1}}{1-5z^{-1}+6z^{-2}} = \dfrac{1-3z^{-1}}{(1-2z^{-1})(1-3z^{-1})}$

$= \dfrac{1}{1-2z^{-1}}$

$h(n) = 2^n u(n)$