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$$E(s) = \frac{1}{s^2} \quad R(s) = \frac{1}{s+a} \quad H(s) = \frac{R(s)}{E(s)} = \frac{s^2}{s+a}$$

$$R(s) = E(s) \cdot H(s) = \frac{s^2}{s+a} = s - a + \frac{a^2}{s+a}$$

$$r(t) = \delta(t) - a\delta'(t) + a^2 e^{-at} u(t)$$

1. 问题可化为特征方程为: $s^2 + 6s + 5 = 0$

$$s_1 = -1 \quad s_2 = -5$$

求 $y_{zi}(t)$

$$y_{zi}(t) = A_1 e^{-t} + B_1 e^{-5t}$$

$$sA_1 + B_1 = y(0^+) = y(0) = 1 \quad A_1 = \frac{7}{4} \quad B_1 = -\frac{3}{4}$$

$$-A_1 - 5B_1 = y'(0^+) = y'(0) = 2$$

$$y_{zi}(t) = \frac{7}{4} e^{-t} - \frac{3}{4} e^{-5t}$$

求 $y_{zs}(t)$

因 $e^{st} = t^2 u(t)$ 可得方程:

$$\frac{d^2 y(t)}{dt^2} + 6 \frac{dy(t)}{dt} + 5y(t) = (2t + 3t^2) u(t)$$

特解 $y^* = at^2 + bt + c$ 代入方程:

$$2a + 12at + 6b + 5at^2 + 10bt + 5c = 2t + 3t^2$$

$$a = \frac{3}{5} \quad b = -\frac{26}{45} \quad c = -\frac{6}{15}$$

$$y_{zs}(t) = A_2 e^{-t} + B_2 e^{-5t} + \frac{3}{5} t^2 - \frac{26}{45} t + \frac{6}{15}$$

没有 s 及其导数, $y(t)$ 在 0 处连续

$$\begin{cases} A_2 + B_2 + \frac{6}{15} = 0 \\ -A_2 - 5B_2 - \frac{26}{45} = 0 \end{cases} \quad \text{得} \quad A_2 = \frac{25}{15} = \frac{5}{3} \quad B_2 = -\frac{31}{15}$$

$$y_{zs}(t) + y_{zi}(t) = \frac{1}{5} e^{-t} - \frac{31}{15} e^{-5t} + \frac{3}{5} t^2 - \frac{26}{45} t + \frac{6}{15}$$

$$y(t) = \frac{39}{10} e^{-t} - \frac{499}{500} e^{-5t} + \frac{3}{5} t^2 - \frac{26}{45} t + \frac{6}{15}$$

$$y(t) = \underbrace{\frac{39}{10} e^{-t} - \frac{499}{500} e^{-5t}}_{\text{自由响应}} + \underbrace{\frac{3}{5} t^2 - \frac{26}{45} t + \frac{6}{15}}_{\text{强迫响应}}$$

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2. 问题

$$\frac{dy(t)}{dt} + 6 \frac{dy(t)}{dt} + 5y(t) = \delta'(t) + 3\delta(t)$$

利用冲激匹配法求 $y(t)$ 及 $y'(t)$

$$y'(t) = a\delta'(t) + b\delta(t) + c u(t)$$

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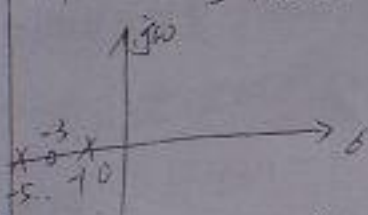
$$a = 1 \quad b + 6a = 3 \quad b = -3$$

$$\text{设 } y(t) = A e^{-t} + B e^{-5t}$$

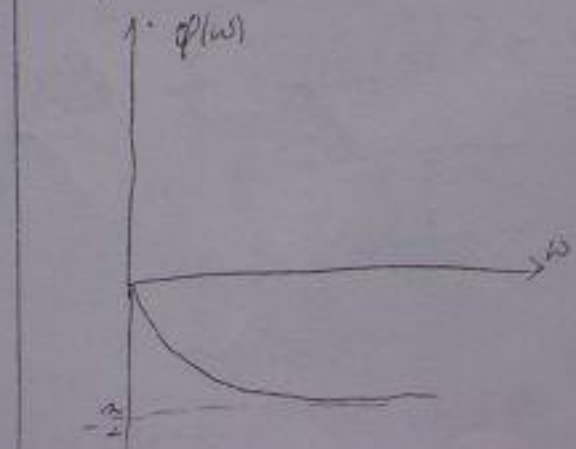
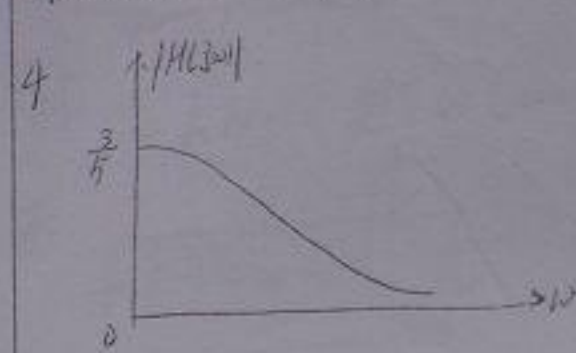
$$\begin{cases} A + B = a = 1 \\ -A - 5B = -3 \end{cases} \quad A = \frac{1}{2} \quad B = \frac{1}{2}$$

$$y(t) = \frac{1}{2} e^{-t} + \frac{1}{2} e^{-5t}$$

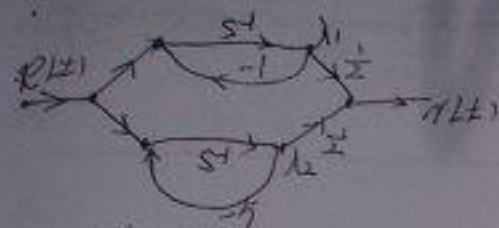
$$H(s) = \frac{s+3}{s^2+6s+5} = \frac{s+3}{(s+1)(s+5)}$$



极点均位于 s 域左平面: 系统稳定



$$h. H(s) = \frac{s+3}{(s+1)(s+5)} = \frac{\frac{1}{4}}{s+1} + \frac{\frac{1}{4}}{s+5}$$



$$\begin{cases} \lambda_1' t = \lambda_1 t + e(t) \\ \lambda_2' t = \lambda_2 t + e(t) \end{cases}$$

$$y(t) = \frac{1}{2} \lambda_1 t + \frac{1}{2} \lambda_2 t$$

$$\begin{bmatrix} \lambda_1' t \\ \lambda_2' t \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \lambda_1 t \\ \lambda_2 t \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} e(t)$$

$$y(t) = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} \lambda_1 t \\ \lambda_2 t \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad C = \left(\frac{1}{2}, \frac{1}{2} \right) \quad D = 0$$

三. 解:

$$y(t) = e(t) * h_1(t) * h_2(t) * h_3(t)$$

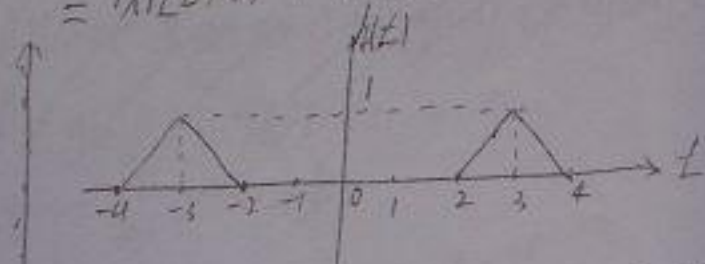
$$h_1(t) = \delta(t) * h_1(t) * h_1(t) * h_1(t)$$

$$= (\delta(t) * h_1(t)) * [\delta(t+4) + \delta(t-2)]$$

$$h_2(t) * h_3(t) = t u(t) + (2-t) u(t-2) - t u(t-2)$$

$$h_3(t) = x_1(t) * [\delta(t+4) + \delta(t-2)]$$

$$= x_1(t+4) + x_1(t-2)$$



$$\begin{aligned} h(t) &= [t+4][u(t+4) - u(t+3)] + (-2+t)[u(t+3) - u(t+2)] \\ &\quad + (t-2)[u(t-2) - u(t-3)] + (-t+4)[u(t-3) - u(t-4)] \end{aligned}$$

$$\begin{aligned} &= \int_{-4}^{-2} (t+4)e^{-j\omega t} dt + \int_{-2}^{-3} (t-2)e^{-j\omega t} dt \\ &\quad + \int_{2}^{3} (t-2)e^{-j\omega t} dt + \int_{3}^{4} (4-t)e^{-j\omega t} dt \\ &= -\frac{(e^{-j\omega} - 1)^2}{\omega^2} (e^{4j\omega} + e^{-2j\omega}) \end{aligned}$$

直接利用 $H_1(j\omega) = H_1(j\omega) \cdot H_2(j\omega)$

四.

1.

$$f_T(t) = f(t) \cdot \delta_T(t) = f(t) \sum_{n=-\infty}^{+\infty} \delta(t - nT)$$

$$F_T(\omega) = \frac{1}{T} F(\omega) * \delta_T(\omega)$$

$$= \frac{1}{T} F(\omega) * \left(\frac{2\pi}{T} \right) \sum_{n=-\infty}^{+\infty} \delta(\omega - n\omega_1) \left(\frac{1}{T} - \frac{2\pi}{T} \right)$$

$$= \frac{1}{T} \sum_{n=-\infty}^{+\infty} F(\omega - n\omega_1)$$

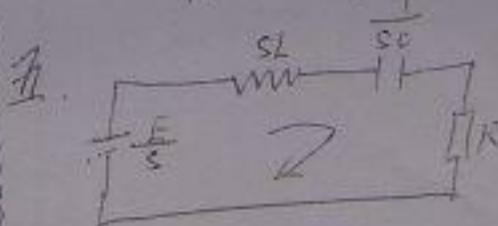
设 $|\omega| > \omega_m$ 时 $|F(\omega)| = 0$

则无失真恢复 $F(\omega)$ 的条件是 $\omega_1 \geq \omega_m$

所通过滤波器的特性

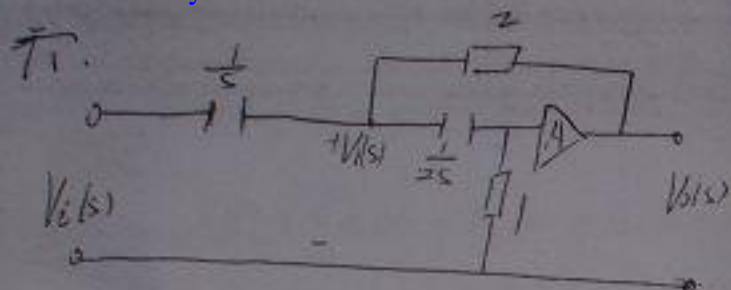
$$\begin{cases} H(\omega) = K & |\omega| \leq \omega_m \\ 0 & |\omega| > \omega_m \end{cases}$$

$$\text{时域: } \delta_a h(t) = \frac{\omega_m}{\pi} \delta_a [\omega_m (t - t_0)]$$



$$\text{问题: } \frac{E}{s} = I(s) \left(sL + \frac{1}{sC} + R \right)$$

$$\begin{aligned} I(s) &= \frac{E}{s} \cdot \frac{1}{s + \frac{2}{s} + 2} \\ &= \frac{E}{s} \cdot \frac{s}{s^2 + 2s + 2} \\ &= \frac{E}{(s+1)^2 + 1} \end{aligned}$$



1. 问题:

$$\begin{cases} \frac{V_o(s) - V_i(s)}{\frac{1}{s}} = \frac{V_i(s) - V_o(s)}{2} + \frac{V_i(s)}{\frac{1}{2s} + 1} \end{cases}$$

$$\frac{V_o(s)}{\frac{1}{2s} + 1} - 1 = V_o(s)$$

$$\text{得: } \frac{V_o(s)}{V_i(s)} = \frac{1}{2} \cdot \frac{s}{4s^2 + 2s + 2s + 1 + (\frac{1}{2s} - \frac{1}{2})}$$

$$= \frac{4As^2}{4s^2 + (8 - 2A)s + 1}$$

若要系统稳定则 $8 - 2A \geq 0 \quad A \leq 4$

$A = 4$ 时

$$\text{此时: } \frac{16As^2}{4s^2 + 1} = 4 - \frac{4}{s^2 + 1}$$

$$f_L(t) = 4\delta(t) - (\sin \frac{1}{2}t) u(t)$$

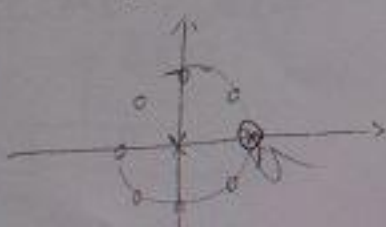
4.

$$\begin{aligned} & (1) y(n) + a^0 y(n-1) + a^2 y(n-2) + a^4 y(n-3) + a^6 y(n-4) \\ & + a^8 y(n-5) + a^{10} y(n-6) \\ & y(n) = \sum_{k=0}^{\infty} a^k x(n-k) \end{aligned}$$

$$2. H(z) = \sum_{k=0}^{\infty} a^k z^{-k} = \frac{1 - a^8 z^{-8}}{1 - az^{-1}}$$

$$3. h(n) = a^n [u(n) - u(n-8)]$$

4.



5.

$$y(n) = h(n) * x(n)$$

利用乘法原理

$$1 \quad a \quad a^2 \quad a^3 \quad a^4 \quad a^5 \quad a^6 \quad a^7$$

$$1 \quad 1 \quad 1$$

$$1 \quad a \quad a^2 \quad a^3 \quad a^4 \quad a^5 \quad a^6 \quad a^7$$

$$1 \quad a \quad a^2 \quad a^3 \quad a^4 \quad a^5 \quad a^6 \quad a^7$$

$$1 \quad a \quad a^2 \quad a^3 \quad a^4 \quad a^5 \quad a^6 \quad a^7$$

利用乘法原理

$$\frac{1}{1-a^8 z^{-8}} = \sum_{n=0}^{\infty} a^{8n} z^{-8n}$$

$$\text{可得 } y(n) = \sum_{k=0}^{\infty} \{1, a^8, a^{16}, a^{24}, a^{32}, a^{40}, a^{48}, a^{56}, a^{64}, a^{72}, a^{80}, a^{88}, a^{96}, a^{104}, a^{112}, a^{120}, a^{128}, a^{136}, a^{144}, a^{152}, a^{160}, a^{168}, a^{176}, a^{184}, a^{192}, a^{200}, a^{208}, a^{216}, a^{224}, a^{232}, a^{240}, a^{248}, a^{256}, a^{264}, a^{272}, a^{280}, a^{288}, a^{296}, a^{304}, a^{312}, a^{320}, a^{328}, a^{336}, a^{344}, a^{352}, a^{360}, a^{368}, a^{376}, a^{384}, a^{392}, a^{400}, a^{408}, a^{416}, a^{424}, a^{432}, a^{440}, a^{448}, a^{456}, a^{464}, a^{472}, a^{480}, a^{488}, a^{496}, a^{504}, a^{512}, a^{520}, a^{528}, a^{536}, a^{544}, a^{552}, a^{560}, a^{568}, a^{576}, a^{584}, a^{592}, a^{600}, a^{608}, a^{616}, a^{624}, a^{632}, a^{640}, a^{648}, a^{656}, a^{664}, a^{672}, a^{680}, a^{688}, a^{696}, a^{704}, a^{712}, a^{720}, a^{728}, a^{736}, a^{744}, a^{752}, a^{760}, a^{768}, a^{776}, a^{784}, a^{792}, a^{800}, a^{808}, a^{816}, a^{824}, a^{832}, a^{840}, a^{848}, a^{856}, a^{864}, a^{872}, a^{880}, a^{888}, a^{896}, a^{904}, a^{912}, a^{920}, a^{928}, a^{936}, a^{944}, a^{952}, a^{960}, a^{968}, a^{976}, a^{984}, a^{992}, a^{1000}\}$$

11.

$$\begin{aligned} H(z) &= \frac{1 - 3z^{-2}}{1 - 5z^{-1} + 6z^{-2}} = \frac{1 - 3z^{-1}}{(1 - 2z^{-1})(1 - 3z^{-1})} \\ &= \frac{1}{1 - 2z^{-1}} \end{aligned}$$

$$h(n) = 2^n u(n)$$