2005.

一、

1.

① $F\{tu(t)\} = \int_0^{+\infty} t e^{-j\omega t} dt = \frac{1}{-j\omega}[te^{-j\omega t}|_0^{+\infty} - \int_0^{+\infty} e^{-j\omega t}dt] = \frac{1}{-j\omega}\cdot\frac{1}{-j\omega}\cdot(-1)e^{-j\omega t}|_0^{+\infty}$

$= \frac{1}{\omega^2}(0-1) = -\frac{1}{\omega^2}$

② $u(t) \overset{F}{\longleftrightarrow} \pi\delta(\omega) + \frac{1}{j\omega}$

$jtu(t) \overset{F}{\longleftrightarrow} \pi\delta'(\omega) + \frac{-j}{\omega^2}$

$tu(t) \overset{F}{\longleftrightarrow} \underline{j\pi\delta'(\omega) + \frac{-1}{\omega^2}}$

2. $F^{-1}\{\frac{2e^{-j\omega}}{j\omega}\}$

$\frac{1}{j\omega} \overset{F^{-1}}{\longleftrightarrow} u(t) - \frac{1}{2}$

$\frac{2e^{-j\omega}}{j\omega} \overset{F^{-1}}{\longleftrightarrow} 2[u(t-1) - \frac{1}{2}] = 2u(t-1) - 1$

$= \mathrm{sgn}(t-1)$

3. $L\{(t-1)^n u(t-1)\}$

$tu(t) \overset{L}{\longleftrightarrow} \frac{1}{s^2}$

$t^2u(t) \longleftrightarrow \frac{1\cdot 2}{s^3}$

$t^3u(t) \longleftrightarrow \frac{(3)!}{s^4}$

$\vdots$

$t^n u(t) \longleftrightarrow \frac{(n)!}{s^{n+1}}$

$\therefore (t-1)^n u(t-1) \longleftrightarrow \frac{(n)!}{s^{n+1}} e^{-s}$

4. $\frac{1}{s^2} \overset{L^{-1}}{\longleftrightarrow} tu(t)$

$\frac{1}{(s+1)^2} \overset{L^{-1}}{\longleftrightarrow} te^{-t}u(t)$

$\frac{e^{-s}}{(s+1)^2} \overset{L^{-1}}{\longleftrightarrow} \underline{(t-1)e^{-(t-1)}u(t-1)}$

5. $u(n) \overset{Z}{\longleftrightarrow} \frac{1}{1-z^{-1}} = \frac{z}{z-1}$

$nu(n) \overset{Z}{\longleftrightarrow} -z\frac{d}{dz}(\frac{z}{z-1})$

$= \frac{z}{(z-1)^2}$

6.

$\frac{1-z^{-4}}{1+z^{-1}} = \frac{(1+z^{-2})(1+z^{-1})(1-z^{-1})}{1+z^{-1}} = 1-z^{-1}+z^{-2}-z^{-3}$

$\frac{1-z^{-4}}{1+z^{-1}} \overset{Z^{-1}}{\longleftrightarrow} \delta(n) - \delta(n-1) + \delta(n-2) - \delta(n-3)$

7.

$DTFT\{(\frac{1}{2})^n u(n)\} = \sum_{n=-\infty}^{+\infty} (\frac{1}{2})^n u(n) e^{-j\omega n}$

$= \sum_{n=0}^{+\infty}(\frac{1}{2})^n e^{-j\omega n} = \frac{1}{1-\frac{1}{2} e^{-j\omega}} = \frac{2}{2-e^{-j\omega}}$

8. $X(e^{j\omega}) = \begin{cases} 2e^{j\frac{\pi}{2}} & 0\le\omega\le\pi \\ 2e^{-j\frac{\pi}{2}} & -\pi\le\omega\le 0\end{cases}$

$x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}d\omega$

$= \frac{1}{2\pi}\int_{-\pi}^{0} -2j\, e^{j\omega n}d\omega + \frac{1}{2\pi}\int_0^{\pi} 2j\, e^{j\omega n} d\omega$

$= \frac{-1}{\pi n} e^{j\omega n}|_{-\pi}^{0} + \frac{1}{\pi n} e^{j\omega n}|_0^{\pi}$

$= \frac{-1}{\pi n}(1-\cos n\pi) + \frac{1}{\pi n}(\cos n\pi - 1)$

$= \frac{2(\cos n\pi - 1)}{n\pi}$

9.

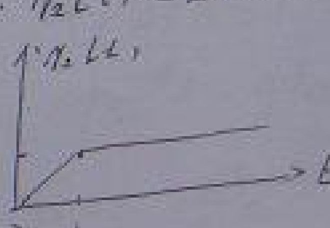
$e_1(t) = \delta(t)$ $E_1(s) = 1$

$r_1(t) = u(t)$ $R_1(s) = \frac{1}{s}$

$E_2(s) = \frac{1-e^{-s}}{s}$

$R_2(s) = \frac{1}{s}\cdot\frac{1-e^{-s}}{s} = \frac{1-e^{-s}}{s^2}$

$\therefore r_2(t) = tu(t) - (t-1)u(t-1)$



三.

解 $H(j\Omega) = \pi\delta(\Omega) - \frac{1}{j\Omega} - \pi\delta(\Omega) + \frac{e^{-j5\Omega}}{j\Omega}$

$R_{zs}(j\Omega) = H(j\Omega)\cdot E(j\Omega)$

$= \left[\frac{e^{-j5\Omega}-1}{j\Omega}\right]\cdot[\delta(\Omega)+\delta(\Omega-2)+\delta(\Omega-5)]$

$= -2\delta(\Omega) + \frac{-\delta(\Omega-2)}{j2} + \frac{e^{-j25}-1}{j5}\delta(\Omega-5)$

$r_{zs}(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} R_{zs}(j\Omega)\, e^{j\Omega t}\, d\Omega$

$= \frac{1}{2\pi}\int_{-\infty}^{+\infty}\left[-2\delta(\Omega) - \frac{\delta(\Omega-2)}{j2} + \frac{e^{-j25}-1}{j5}\delta(\Omega-5)\right]e^{j\Omega t}\, d\Omega$

$= \frac{1}{2\pi}\left[-2 - \frac{e^{j2t}}{j2} + \frac{e^{-j25}-1}{j5}e^{j5t}\right]$

$= \frac{1}{2\pi}\left[-2 - \frac{e^{j2t}}{j2} + \frac{e^{-j25}-1}{j5}e^{j5t}\right]$

$e(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}[\delta(\Omega)+\delta(\Omega-2)+\delta(\Omega-5)]\, e^{j\Omega t}\, d\Omega$

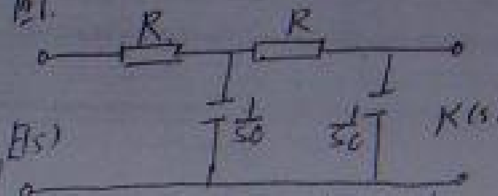
$= \frac{1}{2\pi}\left[1+e^{j2t}+e^{j5t}\right]$

2、5没有公约数 ∴ $e(t)$不是周期信号

$r_{zs}(t)$也不是周期信号

此题有疑问.

四.



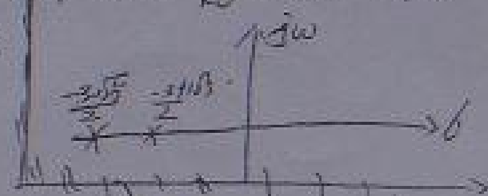
1. $H(s) = \frac{R(s)}{E(s)} = \frac{\frac{1}{sC}}{R+\frac{1}{sC}}\cdot\frac{\frac{\frac{1}{sC}(R+\frac{1}{sC})}{\frac{1}{sC}+R+\frac{1}{sC}}}{R+\frac{\frac{1}{sC}(R+\frac{1}{sC})}{\frac{1}{sC}+R+\frac{1}{sC}}}$

$= \frac{1}{s^2(RC)^2+3sRC+1} = \frac{\frac{1}{R^2C^2}}{s^2+3s\frac{1}{RC}+\frac{1}{R^2C^2}}$

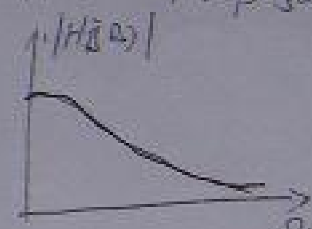
RC均是大于0的，$H(s)$的极点都在s平面的左半平面 ∴系统是稳定的.

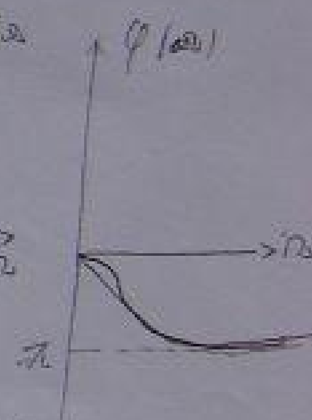
$H(s) = \frac{1}{R^2C^2}\cdot\frac{1}{\left(s-\frac{-3+\sqrt5}{2RC}\right)\left(s-\frac{-3-\sqrt5}{2RC}\right)}$

方程: $r''(t)+\frac{3}{RC}r'(t)+\frac{r(t)}{(RC)^2} = e(t)$



2. $H(j\Omega) = H(s)|_{s=j\Omega}$





不满足不失真条件.

3. 由题系统零输入方程:

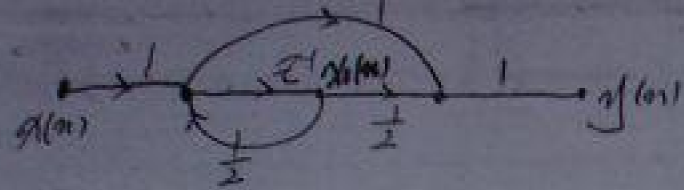
$r''(t)+3r'(t)+r(t)=0$

$r_{zi}(t) = Ae^{\frac{-3+\sqrt5}{2}t}+Be^{\frac{-3-\sqrt5}{2}t}$

则 $\begin{cases} A+B=1 \\ \frac{-3+\sqrt5}{2}A - \frac{3+\sqrt5}{2}B = 1 \end{cases}$ $A=\frac{1+\sqrt5}{2}$ $B=\frac{1-\sqrt5}{2}$

$r_{zi}(t) = \frac{1+\sqrt5}{2}e^{\frac{-3+\sqrt5}{2}t}+\frac{1-\sqrt5}{2}e^{\frac{-3-\sqrt5}{2}t}$

五.



1. $\begin{cases} x(n')+\frac{1}{2}x_1(n') = x_1(n) \\ x(n)+\frac{1}{2}x_1(n) = y(n) \end{cases}$

$x_1(n) = 2[y(n)-x(n)]$

$\frac{1}{2}x_1(n-1) = y(n-1) - x(n-1)$

$\therefore x(n-1) \cancel{+} y(n-1) - x(n-1) = 2y(n) - 2x(n)$

$2y(n) - y(n-1) = 2x(n)$

2. $H(z) = \frac{2}{2-z^{-1}} = \frac{1}{1-\frac{1}{2}z^{-1}}$

$h(n) = \left(\frac{1}{2}\right)^n u(n)$

3. $y(-1)=1$ 时 $2y(n)-y(n-1)=0$

$y(0)=\frac{1}{2}$ $2\alpha-1=0$ $\alpha=\frac{1}{2}$

$y_{zi}(n) = A\left(\frac{1}{2}\right)^n u(n)$

$A=\frac{1}{2}$ $y_{zi}(n) = \left(\frac{1}{2}\right)^{n+1} u(n)$

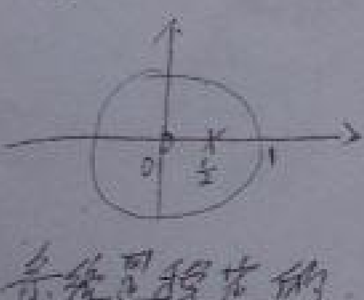
4. $x(n) = u(n)$

$X(z) = \frac{1}{1-z^{-1}}$

$Y(z) = X(z)H(z) = \frac{1}{1-z^{-1}} \cdot \frac{1}{1-\frac{1}{2}z^{-1}} = \frac{2}{1-z^{-1}} + \frac{-1}{1-\frac{1}{2}z^{-1}}$

$\therefore y_{zs}(n) = 2u(n) - \left(\frac{1}{2}\right)^n u(n)$

5.



$H(z) = \frac{z}{z-\frac{1}{2}}$

由于极点都位于单位圆内

$\therefore$ 系统是稳定的